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Intrinsic geometry of the $N = 1$ supersymmetric Yang–Mills theory

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Abstract. The $N = 1$ supersymmetric Yang–Mills theory is formulated analogously to the minimal $N = 1$ supergravity in the Ogievetsky–Sokatchev approach. The intrinsic superspace geometry of the $N = 1$ Yang–Mills is shown to be the complex geometry of embedding of the real superspace $\mathbb{R}^{4|4} = \{x^m, \theta^\mu, \bar{\theta}^\mu = (\theta^\mu)^\dagger\}$ into the extended complex one $\mathbb{C}^{4+M|2} = \{x^m_L, \theta^\mu_L = \theta^\mu, \varphi^i_L\}$, $(i = 1, \ldots, M)$, φ^i_L being local coordinates on the group G^c, the complexification of gauge group G, and $M = \dim G$. The $N = 1$ Yang–Mills prepotential is identified with $\operatorname{Im} \varphi^i_L$ restricted to the hypersurface $\mathbb{R}^{4|4}$. It takes values in the coset G^c/G, so the $N = 1$ Yang–Mills theory can be interpreted as a generalised nonlinear σ model. The corresponding Cartan forms are defined and they are applied to the construction of relevant geometric objects. We discuss also some new possibilities following from the suggested formulation of the theory.

1. Introduction

In superspace, supersymmetric Yang–Mills and supergravity theories reveal quite different geometric structures in comparison with those seen at the component level. Standard gauge potentials in superspace carry too many degrees of freedom even in a fixed gauge and, for this reason, cannot serve as the fundamental quantities. In any self-contained superfield gauge theory they are composite objects constructed from a lesser number of unconstrained superfields, prepotentials. The prepotentials proved to be a very useful concept; being directly related to the physical field content of a given theory, they provide an adequate realisation of its minimal invariance group and hence can be considered as natural carriers of the corresponding intrinsic superspace geometry.

At present, the complete prepotential formulations exist for the $N = 1$ Yang–Mills theory (Ferrara and Zumino 1974, Salam and Strathdee 1974) and $N = 1$ supergravity (Ogievetsky and Sokatchev 1977, 1978a, b, Siegel and Gates 1979) and, at the linearised level, for their $N = 2$ counterparts (Mezincescu 1979, Gates and Siegel 1982)†. The standard strategy to search for prepotentials is as follows. One starts with the ordinary differential geometry in superspace (see e.g. Wess 1981) and then solves proper constraints on covariant strengths, curvatures, torsions, etc. It is not so easy to guess what are the adequate constraints in one or another specific case, because of the lack of a general procedure (though some heuristic principles were suggested

† The quantities suggested by Sokatchev (1981) as the prepotentials of complete $N = 2$ supergravity seem not to be true ones as they are still subjected to certain constraints.

here, e.g. the requirement of preserving certain representations of flat supersymmetry in the curved case (Gates *et al* 1980)).

Another approach, which seems to be of a greater universality, proceeds directly from exposing the minimal invariance group and intrinsic superspace geometry of a given theory. Once these are established, relevant prepotentials are expected to arise naturally within this framework as objects with a clear group and geometric meaning. Such a program has been carried out only for the $N=1$ supergravity as yet. Ogievetsky and Sokatchev (1978a, b, 1980a, b) have shown that the geometry underlying the minimal $N=1$ supergravity is the complex geometry of the real superspace $\mathbb{R}^{4|4}=\{x^m, \theta^\mu, \bar\theta^{\dot\mu}\}$ embedded as a hypersurface into the complex chiral superspace $\mathbb{C}^{4|2}=\{x_L^m, \theta_L^\mu\}$. The geometric role of the corresponding prepotential $H^m(x,\theta,\bar\theta)$ is to specify this embedding:

$$\operatorname{Re} x_L^m = x^m, \qquad \theta_L^\mu=\theta^\mu, \qquad (\theta_L^\mu)^\dagger \equiv \bar\theta_R^{\dot\mu}=\bar\theta^{\dot\mu}, \qquad \operatorname{Im} x_L^m = H^m(x,\theta,\bar\theta). \quad (1.1)$$

The minimal invariance group of $N=1$ supergravity is the supergroup of general analytic coordinate transformations of $\mathbb{C}^{4|2}$ (its divergenceless subgroup in the Einstein case). By the identification (1.1), this supergroup has a natural realisation on $\{x^m, \theta^\mu, \bar\theta^{\dot\mu}, H^n(x,\theta,\bar\theta)\}$. In the case of non-minimal $N=1$ supergravities, $\mathbb{R}^{4|4}$ is embedded into a larger complex superspace $\mathbb{C}^{4|4}$ having two spinor dimensions in addition (Sokatchev 1981). These extra spinor coordinates, being restricted to $\mathbb{R}^{4|4}$, constitute together with $H^m(x,\theta,\bar\theta)$ the full set of relevant prepotentials (Siegel and Gates 1979). It is essential that in both cases, minimal and non-minimal, the prepotentials appear primarily as coordinates of certain complex superspaces.

It is unknown which superfields play the role of prepotentials in gauge theories with $N=2$ (except for $N=2$ electrodynamics) and which geometries are associated with them (an off-shell manifestly supersymmetric superfield formulation is constructed as yet only for $N=2$ Yang–Mills (Grimm *et al* 1978)). No clear geometric interpretation exists even for the $N=1$ Yang–Mills prepotential $V^i(x,\theta,\bar\theta)$ (i is the index of the adjoint representation of the gauge group). At the same time, in order to unmask the minimal geometric structure of higher-N gauge theories (and supergravities as well) it seems necessary first to understand clearly the geometry of the text-book $N=1$ case.

This analysis is performed in the present paper†. We demonstrate that the intrinsic superspace geometry of the $N=1$ Yang–Mills theory reveals a close similarity to that of minimal $N=1$ supergravity. We start with the complexification G^c of the gauge group G and define the extended chiral superspace $\mathbb{C}^{4+M|2}=\{x_L^m, \varphi_L^i, \theta_L^\mu\}$, ($i=1,\ldots,M$), where $M=\dim G$ and φ_L^i are local complex coordinates on G^c. The $N=1$ Yang–Mills theory turns out to be associated with the dynamics of embedding of $\mathbb{R}^{4|4}$ into $\mathbb{C}^{4+M|2}$. The $N=1$ prepotential $V^i(x,\theta,\bar\theta)$ coincides with $\operatorname{Im}\varphi_L^i$ restricted to $\mathbb{R}^{4|4}$ and is introduced by equation (2.11*a*) analogous to the last of equations (1.1). This superfield has a simple meaning: it parametrises the coset space G^c/G and hence specifies the position of the hypersurface $\mathbb{R}^{4|4}$ relative to the G^c/G directions in $\mathbb{C}^{4+M|2}$. $\operatorname{Re}\varphi_L^i$ remains arbitrary and does not influence the dynamics. As a result, $\mathbb{C}^{4+M|2}$ actually reduces to the quotient $\mathbb{C}^{4+M|2}/G$. The fact that $V^i(x,\theta,\bar\theta)$ takes values in the coset G^c/G allows one to interpret the $N=1$ Yang–Mills theory as a generalised nonlinear σ model. Hence, the powerful method of Cartan differential forms (Callan

† Some of the results presented have already been published as a letter (Ivanov 1982).

et al 1969, Volkov 1969, 1973, Ogievetsky 1974) may be applied to the construction of relevant invariants and other geometric objects.

The organisation of this paper is as follows. In § 2 we present the geometric derivation of the $N = 1$ Yang–Mills prepotential. This is similar for the cases of rigid and local supersymmetries. In § 3, the corresponding Cartan forms are defined and it is explained how to construct from them standard geometric characteristics of the $N = 1$ Yang–Mills theory which automatically respect the conventional kinematic constraints (Sohnius 1978). We begin with the case of flat geometry on $\mathbb{R}^{4|4}$ and then extend our study to the case of couplings with $N = 1$ supergravity. In the conclusion we indicate some consequences of the proposed geometric picture and make an attempt to realise what would be the analogue of the complex group G^c in the $N = 2$ Yang–Mills theory.

2. The geometric derivation of the $N = 1$ Yang–Mills prepotential

2.1.

We begin with defining the complex group G^c. It is uniquely constructed by the initial gauge group G. Let G be a compact M-dimensional group with generators T^i, $(i = 1, \ldots, M)$. In the basis where T^i are Hermitian they satisfy the commutation relations

$$[T^i, T^k] = \mathrm{i}c^{ikl}T^l, \tag{2.1}$$

c^{ikl} being real totally skew-symmetric structure constants. G^c is defined as an M-dimensional complex group with M complex generators T^i_{L} which constitute, together with their conjugates $T^i_{\mathrm{R}} \equiv (T^i_{\mathrm{L}})^\dagger$, the following Lie algebra:

$$[T^i_{\mathrm{L}}, T^k_{\mathrm{L}}] = \mathrm{i}c^{ikl}T^l_{\mathrm{L}}, \qquad [T^i_{\mathrm{R}}, T^k_{\mathrm{R}}] = \mathrm{i}c^{ikl}T^l_{\mathrm{R}}, \tag{2.2a}$$

$$[T^i_{\mathrm{L}}, T^k_{\mathrm{R}}] = 0, \tag{2.2b}$$

or, in terms of $2M$ Hermitian generators

$$[T^i, T^k] = \mathrm{i}c^{ikl}T^l, \qquad [T^i, A^k] = \mathrm{i}c^{ikl}A^l, \qquad [A^i, A^k] = -\mathrm{i}c^{ikl}T^l, \tag{2.3}$$

where

$$T^i = T^i_{\mathrm{L}} + T^i_{\mathrm{R}}, \qquad A^k = \mathrm{i}(T^k_{\mathrm{L}} - T^k_{\mathrm{R}}). \tag{2.4}$$

Besides, the group parameters associated with T^i_{L} and T^i_{R} are assumed to be mutually conjugated. The latter is equivalent to the requirement that the corresponding parameters in the Hermitian basis (2.3), (2.4) are real. So, G^c can equivalently be defined as the $2M$-dimensional group over the field of real numbers with the Lie algebra formed by generators T^i, A^k satisfying the relations (2.3). In particular, if $G = \mathrm{SU}(n)$ then $G^c = \mathrm{SL}(n, C)$. It follows from (2.2) that G^c has (formally) the structure of the direct product $G^c = G_{\mathrm{L}} \times G_{\mathrm{R}}$ with G_{L} and $G_{\mathrm{R}} = (G_{\mathrm{L}})^\dagger$ generated, respectively, by T^i_{L} and T^i_{R}. The initial group G with generators T^k appears as a diagonal in this product, and the remaining generators A^k span the real M-dimensional coset space G^c/G. As is implied by the relations (2.3), the latter is symmetric.

It is worth noting that G^c is non-compact; e.g. the last commutator in (2.3) differs in sign from the analogous commutator in the algebra of ordinary compact chiral extension $G\times G$ of G. Due to non-compactness of G^c, any of its unitary representations is infinite dimensional. For this reason, generators T^i, A^k may be simultaneously chosen to be Hermitian (T^i_L, T^i_R mutually conjugated) provided they are realised by certain infinite-dimensional matrices. Our conclusions do not depend on the choice of representation, so in what follows we may regard T^i, A^k as Hermitian without loss of generality.

Let us treat G^c as a Riemannian manifold and introduce, in a vicinity of its identity element, local coordinates $\varphi^i_L, \varphi^i_R = (\varphi^i_L)^\dagger$, using for definiteness the exponential parametrisation of G^c:

$$\begin{aligned} g^c(\varphi_L, \varphi_R) &= g_L(\varphi_L) g_R(\varphi_R) = \exp(i\varphi^k_L T^k_L)\exp(i\varphi^k_R T^k_R) \\ &= \exp[i(\operatorname{Re}\varphi^k_L T^k + \operatorname{Im}\varphi^k_L A^k)]. \end{aligned} \tag{2.5}$$

Note that $\operatorname{Re}\varphi^k_L$ and $\operatorname{Im}\varphi^k_L$ parametrise, respectively, the subgroup G and the coset G^c/G. Now we define the superspace $\mathbb{C}^{4+M|2}$ playing the fundamental role in further consideration. It is the direct sum of ordinary chiral $N=1$ superspace $\mathbb{C}^{4|2} = \{x^m_L, \theta^\mu_L\}$ and the group G_L regarded as a complex M-dimensional manifold:

$$\mathbb{C}^{4+M|2} = \{x^m_L, \theta^\mu_L, \varphi^i_L\} = \mathbb{C}^{4|2} \oplus G_L. \tag{2.6}$$

$\mathbb{C}^{4|2}$ may be flat or curved, depending on whether the rigid or local supersymmetry is dealt with. Since the left and right superspace coordinates x^m_L, θ^μ_L and $x^m_R = (x^m_L)^\dagger$, $\bar\theta^\mu_R = (\theta^\mu_L)^\dagger$ are related by P-parity, it is natural to accept the same convention for φ^i_L, φ^k_R:

$$\varphi^i_L \overset{P}{\leftrightarrow} \varphi^i_R, \qquad \operatorname{Re}\varphi^i_L \overset{P}{\rightarrow} \operatorname{Re}\varphi^i_L, \qquad \operatorname{Im}\varphi^i_L \overset{P}{\rightarrow} -\operatorname{Im}\varphi^i_L. \tag{2.7}$$

Correspondingly, if, e.g., T^i are scalars, A^k must be pseudoscalars:

$$T^i_L \overset{P}{\leftrightarrow} T^i_R, \qquad T^i \overset{P}{\rightarrow} T^i, \qquad A^i \overset{P}{\rightarrow} -A^i. \tag{2.8}$$

Clearly, (2.8) is an automorphism of the algebra (2.2), (2.3).

The next step is to define the action of G^c in $\mathbb{C}^{4+M|2}$. This group can naturally be implemented as the group of left nonlinear translations of the coordinates φ^i_L, φ^k_R:

$$g_L(\lambda_L) g_L(\varphi_L) = g_L(\varphi'_L(\varphi_L, \lambda_L)), \qquad g_R(\lambda_R) g_R(\varphi_R) = g_R(\varphi'_R(\varphi_R, \lambda_R)) \tag{2.9}$$

where $\lambda^i_L, \lambda^i_R = (\lambda^i_L)^\dagger$ are group parameters. To promote global G^c-transformations to the local ones, we assume that λ^i_L are arbitrary analytic functions over the superspace $\mathbb{C}^{4|2}$:

$$\lambda^i_L = \lambda^i_L(x_L, \theta_L), \qquad \lambda^i_R = (\lambda^i_L)^\dagger = \lambda^i_R(x_R, \bar\theta_R). \tag{2.10}$$

The gauge group thus defined constitutes a semi-direct product with the supergroup realised on x^m_L, θ^μ_L†: the Lie bracket of their two arbitrary transformations is a gauge transformation of the type (2.9). This product is contained as a subgroup in the supergroup of general analytic coordinate transformations of $\mathbb{C}^{4+M|2}$ (to be more exact, in its 'triangular' subgroup which leaves invariant the subspace $\mathbb{C}^{4|2}$). As is implied by the relation (2.2*b*), the left and right components of the gauge group $G^c_{loc} = G_{L\,loc} \times G_{R\,loc}$ commute with each other so that at the initial stage the 'left' and 'right' worlds are entirely disjointed (though conjugated).

† In the flat case, it may be the rigid Poincaré or conformal supergroups, and in the curved case the infinite-dimensional supergroup of $N=1$ supergravity.

2.2.

We wish to show that G^c_{loc} is the invariance group of the $N=1$ Yang–Mills theory and that the latter naturally emerges after extracting a special hypersurface in $\mathbb{C}^{4+M|2}$. This hypersurface is the real superspace $\mathbb{R}^{4|4}=\{x^m, \theta^\mu, \bar\theta^{\dot\mu}\}$ just as in the case of $N=1$ supergravity. An essential difference is that it now possesses purely internal degrees of freedom besides those represented by the axial superfield $H^m(x, \theta, \bar\theta)$, (1.1), because of additional bosonic dimensions in $\mathbb{C}^{4+M|2}$. Accordingly, the embedding conditions (1.1) should be supplemented with $2M$ conditions

$$\text{(a)}\ \operatorname{Im}\varphi^i_{\mathrm{L}}=V^i(x,\theta,\bar\theta), \qquad \text{(b)}\ \operatorname{Re}\varphi^i_{\mathrm{L}}=U^i(x,\theta,\bar\theta), \tag{2.11}$$

where V^i and U^i are real pseudoscalar and scalar superfields. Their transformation properties in G^c_{loc} are uniquely determined by those of φ^i_{L}, φ^k_{R}, (2.9). These superfields span, respectively, the coset space G^c/G and the subgroup G. Hence, they are of the Goldstone type with respect to the corresponding G^c transformations. We want G to be unbroken; then $U^i(x, \theta, \bar\theta)$ should be made to possess no dynamical manifestations. To achieve this, one may proceed just as in standard nonlinear σ models (see e.g. Volkov 1973, Gaillard and Zumino 1981) and require the theory to be invariant under the right gauge G-transformations

$$\exp[\mathrm{i}(U^kT^k+V^kA^k)]\to\exp[\mathrm{i}(U^kT^k+V^kA^k)]\exp(\mathrm{i}\lambda^tT^t) \tag{2.12}$$

where $\lambda^t=\lambda^t(x,\theta,\bar\theta)$ are M real superparameters. Then $U^i(x, \theta, \bar\theta)$ represent purely gauge degrees of freedom. The transformations (2.12) can also be represented in terms of the complex superfields $\varphi^k_{\mathrm{L}}(x,\theta,\bar\theta)=U^k(x,\theta,\bar\theta)+\mathrm{i}V^k(x,\theta,\bar\theta)$, $\varphi^k_{\mathrm{R}}(x,\theta,\bar\theta)=\varphi^{k+}_{\mathrm{L}}(x,\theta,\bar\theta)=U^k(x,\theta,\bar\theta)-\mathrm{i}V^k(x,\theta,\bar\theta)$:

$$\exp(\mathrm{i}\varphi^k_{\mathrm{L}}T^k_{\mathrm{L}})\to\exp(\mathrm{i}\varphi^k_{\mathrm{L}}T^k_{\mathrm{L}})\exp(\mathrm{i}\lambda^tT^t_{\mathrm{L}}), \qquad \exp(\mathrm{i}\varphi^k_{\mathrm{R}}T^k_{\mathrm{R}})\to\exp(\mathrm{i}\varphi^k_{\mathrm{R}}T^k_{\mathrm{R}})\exp(\mathrm{i}\lambda^tT^t_{\mathrm{R}}). \tag{2.12a}$$

From the geometric point of view, the invariance under (2.12) means that different G-directions in $\mathbb{C}^{4+M|2}$ are indistinguishable; the dynamics is required to depend only on the position of the hypersurface $\mathbb{R}^{4|4}$ with respect to directions spanning the coset space G^c/G. In other words, it is the quotient $\mathbb{C}^{4+M|2}/G$ which really enters after allowing for the gauge freedom (2.12).

Upon imposing the natural gauge condition

$$U^i(x,\theta,\bar\theta)=0 \tag{2.13}$$

we are left with M pseudoscalar superfields $V^i(x,\theta,\bar\theta)$ which live in the coset G^c/G and transform under G^c_{loc} according to the generic formula of nonlinear realisations (Coleman *et al* 1969, Volkov 1969, 1973)

$$\exp[\mathrm{i}(\operatorname{Re}\lambda^k_{\mathrm{L}}T^k+\operatorname{Im}\lambda^k_{\mathrm{L}}A^k)]\exp(\mathrm{i}V^kA^k)=\exp(\mathrm{i}V^{k\prime}A^k)\exp[\mathrm{i}K^t(V,\lambda_{\mathrm{L}})T^t] \tag{2.14}$$

with λ^k_{L} as in (2.10). The transformation law of matter superfields $\Phi(x, \theta, \bar\theta)$ can be defined following the general prescriptions of the references cited above:

$$\Phi'(x,\theta,\bar\theta)=\exp(\mathrm{i}K^t\bar T^t)\Phi(x,\theta,\bar\theta) \tag{2.15}$$

where $\bar T^t$ is a proper matrix representation of G-generators (for brevity, indices of the representation are suppressed).

Now, let us demonstrate that the law (2.14) is actually equivalent to the standard transformation law of the $N=1$ Yang–Mills prepotential (Ferrara and Zumino 1974, Salam and Strathdee 1974). To this end, we exploit first the automorphism (2.4) of the algebra (2.3) to rewrite (2.14) in another form:

$$\exp[\mathrm{i}(\mathrm{Re}\,\lambda_{\mathrm{L}}^{k}T^{k}-\mathrm{Im}\,\lambda_{\mathrm{L}}^{k}A^{k})]\exp(-\mathrm{i}V^{t}A^{t})=\exp(-\mathrm{i}V^{t'}A^{t})\exp(\mathrm{i}K^{t}T^{t}). \tag{2.14$'$}$$

The next step is to eliminate the factor $\exp(\mathrm{i}K^{t}T^{t})$ from (2.14) and (2.14′). That yields one more possible form of the transformation of $V^{i}(x,\theta,\bar{\theta})$:

$$\exp[\mathrm{i}(\mathrm{Re}\,\lambda_{\mathrm{L}}^{k}T^{k}+\mathrm{Im}\,\lambda_{\mathrm{L}}^{k}A^{k})]\exp(2\mathrm{i}V^{k}A^{k})\times\exp[-\mathrm{i}(\mathrm{Re}\,\lambda_{\mathrm{L}}^{k}T^{k}-\mathrm{Im}\,\lambda_{\mathrm{L}}^{k}A^{k})]=\exp(2\mathrm{i}V^{t'}A^{t}). \tag{2.16}$$

Finally, passing to the complex generators T_{L}^{i}, T_{R}^{k} (by the formula (2.4)) and taking into account their commutativity we observe that (2.16) is equivalent to the following equation:

$$\exp(\mathrm{i}\lambda_{\mathrm{L}}^{k}T_{\mathrm{L}}^{k})\exp(-2V^{k}T_{\mathrm{L}}^{k})\exp(-\mathrm{i}\lambda_{\mathrm{R}}^{k}T_{\mathrm{L}}^{k})=\exp(-2V^{k'}T_{\mathrm{L}}^{k}) \tag{2.17}$$

(or with T_{R}^{i} instead of T_{L}^{i}). This is just what we are aiming at because T_{L}^{i} fulfil the same commutation relations as T^{i} while the structure of $V^{k'}$ in (2.14) does not depend on a particular choice of generators and is determined solely by their commutation relations.

In fact, the standard form of the $N=1$ prepotential transformation law (with T^{i} in place of T_{L}^{i}) is recovered by substituting for A^{i} in (2.16) its particular representation:

$$\bar{A}_{\mathrm{L}}^{i}=\mathrm{i}\bar{T}^{i}\qquad(\bar{T}_{\mathrm{L}}^{k}=\bar{T}^{k},\ \bar{T}_{\mathrm{R}}^{k}=0). \tag{2.18a}$$

This choice is non-self-conjugated, in accordance with the property that any finite-dimensional representation of the non-compact group G^{c} is non-unitary. Through the identification (2.18*a*) or the conjugated one

$$\bar{A}_{\mathrm{R}}^{k}=-\mathrm{i}\bar{T}^{k}\qquad(\bar{T}_{\mathrm{L}}^{k}=0,\ \bar{T}_{\mathrm{R}}^{k}=\bar{T}^{k}) \tag{2.18b}$$

any representation of G can be extended to that of the whole G^{c}. Then, using the general connection between representations and nonlinear realisations (Coleman *et al* 1969), one may relate any matter superfield with the standard nonlinear transformation law (2.15) to the superfields transforming in $G_{\mathrm{loc}}^{\mathrm{c}}$ linearly according to the representations (2.18*a*), (2.18*b*):

$$\Phi_{\mathrm{L}}=\exp(\mathrm{i}V^{t}\bar{A}_{\mathrm{L}}^{t})\Phi=\exp(-V^{t}\bar{T}^{t})\Phi,\qquad\Phi_{\mathrm{R}}=\exp(\mathrm{i}V^{t}\bar{A}_{\mathrm{R}}^{t})\Phi=\exp(V^{t}\bar{T}^{t})\Phi,$$
$$\Phi_{\mathrm{L}}'=\exp(\mathrm{i}\lambda_{\mathrm{L}}^{t}\bar{T}^{t})\Phi_{\mathrm{L}},\qquad\Phi_{\mathrm{R}}'=\exp(\mathrm{i}\lambda_{\mathrm{R}}^{t}\bar{T}^{t})\Phi_{\mathrm{R}}. \tag{2.19}$$

These relations can be interpreted as describing the transition from the real basis in the group space of G^{c} to its complex left and right bases, in a perfect analogy with the connection between real and complex bases in superspace†. Note that the left and right images of Φ are necessarily complex even if Φ itself is real. However, no actual doubling of degrees of freedom occurs in this case, because the equivalency connection arises only between Φ and the real parts of Φ_{L}, Φ_{R}. The imaginary parts begin from the terms bilinear in V^{k} and Φ. The relations (2.19) were known earlier (Siegel and Gates 1979, Grisaru 1981) but our consideration supplies them with a clear group-theoretical meaning. Note that the substitution of (2.18*a*) or (2.18*b*) in the basic law (2.14) yields the transformation of the $N=1$ Yang–Mills prepotential

† To avoid a possible misunderstanding, we note that no correlation exists between choices of bases in G^{c} and in superspace.

in the form given by Siegel and Gates (1979):

$$\begin{aligned}&\exp(\mathrm{i}\lambda_{\mathrm{L}}^{k}\bar{T}^{k})\exp(-V^{k}\bar{T}^{k})=\exp(-V^{k'}\bar{T}^{k})\exp(\mathrm{i}K^{\prime}{}'\bar{T}^{\prime}),\\&\exp(\mathrm{i}\lambda_{\mathrm{R}}^{k}\bar{T}^{k})\exp(V^{k}\bar{T}^{k})=\exp(V^{k'}\bar{T}^{k})\exp(\mathrm{i}K^{\prime}{}'\bar{T}^{\prime}).\end{aligned}\tag{2.20}$$

Also, the invariance under right gauge G-transformations (2.12) reduces to the well known freedom of complexifying the prepotential:

$$\exp(-V^{i}\bar{T}^{i})\rightarrow\exp(-W^{i}\bar{T}^{i})=\exp(-V^{i}\bar{T}^{i})\exp(\mathrm{i}\lambda^{k}\bar{T}^{k}),\tag{2.21}$$

$$\exp(2V^{i}\bar{T}^{i})=\exp(W^{k+}\bar{T}^{k})\exp(W^{k}\bar{T}^{k}).\tag{2.22}$$

In fact, $W^{i}(x,\theta,\bar{\theta})$, $W^{i+}(x,\theta,\bar{\theta})$ coincide, up to a numerical factor, with the $\mathbb{R}^{4|4}$ restrictions of initial complex group coordinates:

$$W^{i}(x,\theta,\bar{\theta})=-\mathrm{i}\varphi_{\mathrm{L}}^{i}(x,\theta,\bar{\theta}),\qquad W^{i+}(x,\theta,\bar{\theta})=\mathrm{i}\varphi_{\mathrm{R}}^{i}(x,\theta,\bar{\theta}).$$

The relation (2.22) can be looked upon as the invariant definition of $V^{k}(x,\theta,\bar{\theta})$ (the RHS of (2.22) is manifestly invariant with respect to (2.12) or (2.21)).

To summarise, we have derived the $N=1$ Yang–Mills prepotential $V^{i}(x,\theta,\bar{\theta})$ from simple geometric and group principles similar to those constituting the basis of the Ogievetsky–Sokatchev formulation of minimal $N=1$ supergravity. We have started with the extended group G^{c} gauged in a proper way over the $N=1$ chiral superspace $\mathbb{C}^{4|2}$ and have identified $V^{i}(x,\theta,\bar{\theta})$ with the parameters of the coset G^{c}/G. The transformation law of V^{i} has then been deduced by general recipes of group realisations in homogeneous spaces, based merely upon the commutation relations (2.2), (2.3). In previous studies, the transformation rule of V^{i} and V^{i} itself either were simply postulated (Ferrara and Zumino 1974, Salam and Strathdee 1974) or appeared as a solution of proper constraints on covariant strengths (Sohnius 1978). The underlying complex group structure of $N=1$ Yang–Mills remained implicit because the generators of G^{c} appeared always in their particular form (2.18).

2.3.

In the remainder of this section we discuss some peculiarities of transformations (2.14), (2.15). With the restriction to the global case, these transformations display the conventional structure of nonlinear realisations: they are nonlinear if they belong to the coset G^{c}/G $(\operatorname{Re}\lambda_{\mathrm{L}}^{i}=0, \operatorname{Im}\lambda_{\mathrm{L}}^{i}\neq 0)$ and become linear on the subgroup G $(\operatorname{Re}\lambda_{\mathrm{L}}^{i}\neq 0, \operatorname{Im}\lambda_{\mathrm{L}}^{i}=0)$:

$$\delta_{G}V^{k}=c^{klt}V^{l}\operatorname{Re}\lambda_{\mathrm{L}}^{t},\qquad K^{l}(V,\operatorname{Re}\lambda_{\mathrm{L}})=\operatorname{Re}\lambda_{\mathrm{L}}^{l}.\tag{2.23}$$

In the local case, due to the specific form of gauge parameters (2.10), the situation is more complicated. Superfunctions $\operatorname{Re}\lambda_{\mathrm{L}}^{i}$, $\operatorname{Im}\lambda_{\mathrm{L}}^{i}$ are not quite independent, nullifying $\operatorname{Re}\lambda_{\mathrm{L}}^{i}$ reduces $\operatorname{Im}\lambda_{\mathrm{L}}^{k}$ to constants, and *vice versa*. Choosing, e.g., the probe gauge functions to be

$$\operatorname{Re}\lambda_{\mathrm{L}}^{i}|_{\theta=\bar{\theta}=0}=\omega^{i}(x),\qquad \operatorname{Im}\lambda_{\mathrm{L}}^{i}|_{\theta=\bar{\theta}=0}=0\tag{2.24}$$

(these span the subgroup of G^{c}_{loc} preserving the Wess–Zumino gauge) one observes that $\omega^{i}(x)$ enter also into the coefficients of higher θ-monomials in $\operatorname{Re}\lambda_{\mathrm{L}}^{i}$, $\operatorname{Im}\lambda_{\mathrm{L}}^{i}$, because of θ dependence in the formula relating x_{L}^{m} to x^{m} $(x_{\mathrm{L}}^{m}=x^{m}+\mathrm{i}H^{m}(x,\theta,\bar{\theta}))$. For instance, in the flat case $(H^{m}(x,\theta,\bar{\theta})=\theta\sigma^{m}\bar{\theta})$ the whole $\operatorname{Re}\lambda_{\mathrm{L}}^{i}$, $\operatorname{Im}\lambda_{\mathrm{L}}^{k}$ corresponding to the choice (2.24) are

$$\mathrm{Re}\,\lambda^i_{\mathrm{L}} = \omega^i(x) - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta\Box\omega^i(x), \qquad \mathrm{Im}\,\lambda^i_{\mathrm{L}} = \theta\partial\bar\theta\omega^i(x). \tag{2.25}$$

Thus, the local transformations belonging to the subgroup G are necessarily accompanied by certain 'induced' local G^c/G-transformations which depend on derivatives of gauge parameters of G. For this reason, even in the Wess–Zumino gauge

$$V^i_{\mathrm{WZ}}(x, \theta, \bar\theta) = \theta\sigma^m\bar\theta b^i_m(x) + \theta\theta\bar\theta\bar\chi^i(x) + \bar\theta\bar\theta\theta\chi^i(x) + \theta\theta\bar\theta\bar\theta D^i(x)$$

the transformation (2.15) of matter superfields reveals an explicit dependence on gauge fields:

$$K^i_{\mathrm{WZ}} = \omega^i(x) - \tfrac{1}{4}\theta\theta\bar\theta\bar\theta[\Box\omega^i(x) + c^{ikl}b^k_m(x)\partial^m\omega^l(x)] + \mathrm{O}(\omega^2). \tag{2.26}$$

However, one may always redefine Φ so as to bring it to transform in the conventional manner:

$$\Phi \to \tilde\Phi = \exp(\tfrac{1}{4}\theta\theta\bar\theta\bar\theta\partial^m b^k_m \bar T^k)\Phi, \qquad \tilde\Phi' = \exp[\mathrm{i}\omega^k(x)\bar T^k]\tilde\Phi.$$

Finally, we notice that the non-compactness of G^c has no explicit dynamical manifestation at the level of physical components. This is because the G^c symmetry is spontaneously broken from the beginning to the compact symmetry with respect to G and, besides, the Goldstone fields associated with this breaking are purely gauge degrees of freedom (they are contained in the superspin zero part of V^i). In the WZ gauge, the G^c/G transformations of the remaining component fields have the form of ordinary gauge G-transformations and so are completely hidden. On the other hand, in any supersymmetric gauge they appear independently. Thus, the G^c/G invariance can be treated as the consistency condition between ordinary gauge invariance and manifest supersymmetry.

3. The Cartan form analysis of the $N = 1$ Yang–Mills theory

3.1.

It follows from the above consideration that the $N = 1$ Yang–Mills theory, from the group-theoretic point of view, is a kind of generalised nonlinear σ model†. Indeed, V^i takes its values in the coset G^c/G and thus provides the nonlinear realisation of G^c, in a complete analogy, say, with the pion in chiral dynamics which provides the nonlinear realisation of the chiral group $\mathrm{SU}(2)\times\mathrm{SU}(2)$ ($\exp(2\mathrm{i}V^kA^k)$ is nothing but the 'chiral field' on the coset G^c/G). Therefore, invariants and other geometric objects of $N = 1$ Yang–Mills should have an adequate expression in the universal language of Cartan differential forms which is of common use in theories with the nonlinearly realised symmetry (Callan *et al* 1969, Volkov 1969, 1973, Ogievetsky 1974). In the present section, we construct the Cartan forms of the $N = 1$ Yang–Mills theory and show that they provide a convenient general basis for analysing the dynamical structure of this theory. Many empiric tricks applied earlier in constructing invariants, strengths, etc, get a clear group-theoretic meaning in the Cartan form approach. It possesses a great degree of automatism and, we hope, may work in gauge theories with higher N.

† An analogous fact for ordinary gauge theories has been established by Ivanov and Ogievetsky (1976a, b). In the supercase, the similarity with nonlinear σ models is even more transparent and striking.

The basic forms in the present case are spinorial ones, and are introduced by the relations

$$\begin{aligned}\exp(-\mathrm{i}V^kA^k)(\mathscr{D}_\alpha+\mathrm{i}\mathscr{V}^{\mathrm{L}}_\alpha)\exp(\mathrm{i}V^kA^k)&=\mathrm{i}(\omega^t_\alpha A^t+\Omega^t_\alpha T^t)\equiv\mathrm{i}\Omega_\alpha,\\ \exp(-\mathrm{i}V^kA^k)(\bar{\mathscr{D}}_{\dot\alpha}+\mathrm{i}\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha})\exp(\mathrm{i}V^kA^k)&=\mathrm{i}(\bar\omega^t_{\dot\alpha}A^t+\bar\Omega^t_{\dot\alpha}T^t)\equiv\mathrm{i}\bar\Omega_{\dot\alpha}.\end{aligned}\tag{3.1}$$

Here, $\mathscr{D}_\alpha$, $\bar{\mathscr{D}}_{\dot\alpha}$ are ordinary covariant spinor derivatives (they may correspond to the flat as well as curved geometries on $\mathbb{R}^{4|4}$) and $\mathscr{V}^{\mathrm{L}}_\alpha\equiv\mathscr{V}^{\mathrm{L}t}_\alpha T^t_{\mathrm{L}}$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}=(\mathscr{V}^{\mathrm{L}}_\alpha)^+\equiv\bar{\mathscr{V}}^{\mathrm{R}t}_{\dot\alpha}T^t_{\mathrm{R}}$ are spinor connections on the group $G^{\mathrm{c}}_{\mathrm{loc}}$:

$$\begin{aligned}\mathscr{V}^{\mathrm{L}'}_\alpha&=\exp(\mathrm{i}\lambda^k_{\mathrm{L}}T^k_{\mathrm{L}})\mathscr{V}^{\mathrm{L}}_\alpha\exp(-\mathrm{i}\lambda^k_{\mathrm{L}}T^k_{\mathrm{L}})+(1/\mathrm{i})\exp(\mathrm{i}\lambda^k_{\mathrm{L}}T^k_{\mathrm{L}})\mathscr{D}_\alpha\exp(-\mathrm{i}\lambda^k_{\mathrm{L}}T^k_{\mathrm{L}}),\\ \bar{\mathscr{V}}^{\mathrm{R}'}_{\dot\alpha}&=\exp(\mathrm{i}\lambda^k_{\mathrm{R}}T^k_{\mathrm{R}})\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}\exp(-\mathrm{i}\lambda^k_{\mathrm{R}}T^k_{\mathrm{R}}+(1/\mathrm{i})\exp(\mathrm{i}\lambda^k_{\mathrm{R}}T^k_{\mathrm{R}})\bar{\mathscr{D}}_{\dot\alpha}\exp(-\mathrm{i}\lambda^k_{\mathrm{R}}T^k_{\mathrm{R}}).\end{aligned}\tag{3.2}$$

Their role is to compensate the non-commutativity of differential operators $\mathscr{D}_\alpha$, $\bar{\mathscr{D}}_{\dot\alpha}$ on the LHS of (3.1) with elements of gauge groups $G_{\mathrm{L\,loc}}$, $G_{\mathrm{R\,loc}}$, respectively. We shall see later that $\mathscr{V}^{\mathrm{L}}_\alpha$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}$ can be constructed from $V^i(x,\theta,\bar\theta)$ itself.

It is easy to check that under the gauge group (2.14), (3.2), the objects ω^t_α, Ω^t_α and their conjugates exhibit the standard transformation properties of Cartan forms:

$$\begin{aligned}\Omega'_\alpha&=\exp(\mathrm{i}K^tT^t)(\Omega_\alpha-\mathrm{i}\mathscr{D}_\alpha)\exp(-\mathrm{i}K^tT^t),\\ \bar\Omega'_{\dot\alpha}&=\exp(\mathrm{i}K^tT^t)(\bar\Omega_{\dot\alpha}-\mathrm{i}\bar{\mathscr{D}}_{\dot\alpha})\exp(-\mathrm{i}K^tT^t).\end{aligned}\tag{3.3}$$

As follows from (3.3) and the commutation relations (2.3), the quantities ω^t_α, $\bar\omega^t_{\dot\alpha}$ transform homogeneously

$$\begin{aligned}\omega^{t'}_\alpha A^t&=\exp(\mathrm{i}K^lT^l)\omega^t_\alpha A^t\exp(-\mathrm{i}K^lT^l),\\ \bar\omega^{t'}_{\dot\alpha}A^t&=\exp(\mathrm{i}K^lT^l)\bar\omega^t_{\dot\alpha}A^t\exp(-\mathrm{i}K^lT^l),\end{aligned}\tag{3.4}$$

and can be interpreted as gauge-covariant spinor derivatives of the prepotential V^k. The remaining forms Ω^t_α, $\bar\Omega^t_{\dot\alpha}$ are the connections on the coset space G^{c}/G: they transform according to the inhomogeneous law (3.3). These forms define the gauge-covariant spinor derivatives of matter superfields:

$$\nabla_\alpha\Phi=(\mathscr{D}_\alpha+\mathrm{i}\Omega^t_\alpha\bar T^t)\Phi,\qquad \bar\nabla_{\dot\alpha}\Phi=(\bar{\mathscr{D}}_{\dot\alpha}+\mathrm{i}\bar\Omega^t_{\dot\alpha}\bar T^t)\Phi.\tag{3.5}$$

Now, let us come back to the discussion of the status of gauge superpotentials $\mathscr{V}^{\mathrm{L}}_\alpha$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}$. Fortunately, there is no need to associate with them independent degrees of freedom. These superfields can be taken as composite by imposing the manifestly covariant constraints

$$\omega^t_\alpha=\bar\omega^t_{\dot\alpha}=0.\tag{3.6}$$

The equations (3.6) are algebraic with respect to $\mathscr{V}^{\mathrm{L}}_\alpha$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}$, therefore they can easily be solved to give

$$\mathscr{V}^{\mathrm{L}}_\alpha=(1/\mathrm{i})\exp(-2V^tT^t_{\mathrm{L}})\mathscr{D}_\alpha\exp(2V^tT^t_{\mathrm{L}}),\tag{3.7a}$$

$$\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}=(1/\mathrm{i})\exp(2V^tT^t_{\mathrm{R}})\bar{\mathscr{D}}_{\dot\alpha}\exp(-2V^tT^t_{\mathrm{R}})\tag{3.7b}$$

(in deriving (3.7), we have taken advantage of the automorphism (2.8)). Using the transformation of the prepotential in the form (2.17) and in the conjugated one, one may directly check that $\mathscr{V}^{\mathrm{L}}_\alpha$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}$ thus arranged respect the original transformation properties (3.2)†. After substituting (3.7) back into the basic relation (3.1) we are left with the spinor connections on the coset G^c/G expressed solely in terms of $V^i(x, \theta, \bar{\theta})$:

$$\Omega_\alpha = \Omega^i_\alpha T^i = (1/\mathrm{i}) \exp(-V^k T^k) \mathscr{D}_\alpha \exp(V^k T^k),$$
$$\bar{\Omega}_{\dot{\alpha}} = \bar{\Omega}^i_{\dot{\alpha}} T^i = (1/\mathrm{i}) \exp(V^k T^k) \bar{\mathscr{D}}_{\dot{\alpha}} \exp(-V^k T^k). \tag{3.8}$$

These connections are fundamental quantities of which all the geometric characteristics of the theory can be built up: invariants, covariant strengths. This can be done following the standarad procedure (Sohnius 1978, Wess 1981). We find it instructive to repeat the derivation in the context of the proposed geometric interpretation of the theory.

3.2.

Till this point our consideration proceeded in the same way both for rigid and local supersymmetries. Now, we need the explicit form of spinor derivatives $\mathscr{D}_\alpha$, $\bar{\mathscr{D}}_{\dot{\alpha}}$ (to be more precise, it is the explicit form of their mutual anticommutator and commutator with the vector derivative which is really relevant). We begin with the flat case and choose the real basis in superspace $\mathbb{R}^{4|4} = \{x^a, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\}$. In this basis

$$\mathscr{D}_\alpha = \partial/\partial\theta^\alpha - \mathrm{i}(\partial\!\!\!/\bar{\theta})_\alpha, \qquad \bar{\mathscr{D}}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}} + \mathrm{i}(\theta\partial\!\!\!/)_{\dot{\alpha}}, \tag{3.9}$$

$$\{\mathscr{D}_\alpha, \mathscr{D}_\beta\} = \{\bar{\mathscr{D}}_{\dot{\alpha}}, \bar{\mathscr{D}}_{\dot{\beta}}\} = 0, \tag{3.10a}$$

$$\{\mathscr{D}_\alpha, \bar{\mathscr{D}}_{\dot{\beta}}\} = 2\mathrm{i}(\partial\!\!\!/)_{\alpha\dot{\beta}} \equiv 2\mathrm{i}\sigma^a_{\alpha\dot{\beta}}\partial_a. \tag{3.10b}$$

Then, the gauge-covariant spinor derivatives ∇_α, $\bar{\nabla}_{\dot{\alpha}}$, (3.5) satisfy the following commutation relations:

$$\{\nabla_\alpha, \nabla_\beta\} = \mathrm{i}F_{\alpha\beta}, \qquad \{\bar{\nabla}_{\dot{\alpha}}, \bar{\nabla}_{\dot{\beta}}\} = \mathrm{i}\bar{F}_{\dot{\alpha}\dot{\beta}}, \tag{3.11a,b}$$

$$\{\nabla_\alpha, \bar{\nabla}_{\dot{\beta}}\} = 2\mathrm{i}(\partial\!\!\!/ + \Omega)_{\alpha\dot{\beta}} \equiv 2\mathrm{i}\sigma^a_{\alpha\dot{\beta}}\nabla_a, \tag{3.11c}$$

where $F_{\alpha\beta}$, $\bar{F}_{\dot{\alpha}\dot{\beta}}$, $\Omega_{\alpha\dot{\beta}}$ are the G-algebra-valued two-forms expressed in terms of Ω_α, $\bar{\Omega}_{\dot{\alpha}}$ as

$$F_{\alpha\beta} = \mathscr{D}_\alpha\Omega_\beta + \mathscr{D}_\beta\Omega_\alpha + \mathrm{i}\{\Omega_\alpha, \Omega_\beta\}, \tag{3.12}$$

$$\bar{F}_{\dot{\alpha}\dot{\beta}} = \bar{\mathscr{D}}_{\dot{\alpha}}\bar{\Omega}_{\dot{\beta}} + \bar{\mathscr{D}}_{\dot{\beta}}\bar{\Omega}_{\dot{\alpha}} + \mathrm{i}\{\bar{\Omega}_{\dot{\alpha}}, \bar{\Omega}_{\dot{\beta}}\}, \tag{3.13}$$

$$\Omega_{\alpha\dot{\beta}} = \tfrac{1}{2}(\mathscr{D}_\alpha\bar{\Omega}_{\dot{\beta}} + \bar{\mathscr{D}}_{\dot{\beta}}\Omega_\alpha + \mathrm{i}\{\Omega_\alpha, \bar{\Omega}_{\dot{\beta}}\}). \tag{3.14}$$

The covariant strengths $F_{\alpha\beta}$, $\bar{F}_{\dot{\alpha}\dot{\beta}}$ transform in G^c_{loc} homogeneously, according to the same law (3.4) as ω^i_α, $\bar{\omega}^i_{\dot{\alpha}}$. Substitution of the explicit expressions (3.8) for the forms Ω_α, $\bar{\Omega}_{\dot{\alpha}}$ into (3.12), (3.13) yields

$$F_{\alpha\beta} = \bar{F}_{\dot{\alpha}\dot{\beta}} = 0 \tag{3.15}$$

that are just the constraints placed on the strengths in the traditional approach starting from the gauge potentials in superspace (Sohnius 1978, Wess 1981). The quantity

† The possibility of covariant elimination of a number of gauge fields in nonlinear realisations in terms of the Goldstone fields was investigated in detail by Ogievetsky and the author (Ivanov and Ogievetsky 1975). We called it the inverse Higgs phenomenon.

$\Omega_{\alpha\dot\beta}$ is the vector connection

$$\Omega_{\alpha\dot\beta} \equiv \mathrm{i}\sigma^a_{\alpha\dot\beta}\Omega_a = \exp(-\mathrm{i}V^t A^t)(\partial + \mathcal{V})_{\alpha\dot\beta}\exp(\mathrm{i}V^t A^t) \tag{3.16}$$

where

$$\mathcal{V}_{\alpha\dot\beta} \equiv \mathrm{i}\sigma^a_{\alpha\dot\beta}\mathcal{V}_a = \tfrac{1}{2}(\mathscr{D}_\alpha \bar{\mathcal{V}}^{\mathrm{R}}_{\dot\beta} + \bar{\mathscr{D}}_{\dot\beta}\mathcal{V}^{\mathrm{L}}_\alpha + \mathrm{i}\{\mathcal{V}^{\mathrm{L}}_\alpha, \bar{\mathcal{V}}^{\mathrm{R}}_{\dot\beta}\}) = \tfrac{1}{2}(\mathscr{D}_\alpha \bar{\mathcal{V}}^{\mathrm{R}}_{\dot\beta} + \bar{\mathscr{D}}_{\dot\beta}\mathcal{V}^{\mathrm{L}}_\alpha) \tag{3.17}$$

(the commutation relation (2.2b) has been used in passing to the final form of (3.17)). Note that $\mathcal{V}_{\alpha\beta}$ takes its values in the algebra of the whole group G^c while $\Omega_{\alpha\beta}$ takes values in the algebra of G. This can be seen by explicit calculation, with making use of the particular form (3.7) of $\mathcal{V}^{\mathrm{L}}_\alpha$, $\bar{\mathcal{V}}^{\mathrm{R}}_{\dot\alpha}$. The composite gauge superfield $\mathcal{V}_a(x, \theta, \bar\theta)$ transforms under G^c_{loc} according to

$$\mathcal{V}'_a = g^c(\lambda_{\mathrm{L}}, \lambda_{\mathrm{R}})\mathcal{V}_a g^{c^{-1}}(\lambda_{\mathrm{L}}, \lambda_{\mathrm{R}}) + (1/\mathrm{i})g^c(\lambda_{\mathrm{L}}, \lambda_{\mathrm{R}})\partial_a g^{c^{-1}}(\lambda_{\mathrm{L}}, \lambda_{\mathrm{R}}) \tag{3.18}$$

thereby ensuring the standard transformation law for Ω_a:

$$\Omega'_a = \exp(\mathrm{i}K^t T^t)(-\mathrm{i}\partial_a + \Omega_a)\exp(-\mathrm{i}K^t T^t) \tag{3.19}$$

which is quite similar to the laws (3.3). For completeness, we quote the explicit expression of $\Omega_{\alpha\dot\beta}$ in terms of the prepotential V^k:

$$\begin{aligned}\Omega_{\alpha\dot\beta} &= \exp(V^k T^k)[\partial_{\alpha\dot\beta} + (1/2\mathrm{i})\bar{\mathscr{D}}_{\dot\beta}(\exp(-2V^t T^t)\mathscr{D}_\alpha \exp(2V^t T^t))]\exp(-V^k T^k)\\ &= \exp(-V^k T^k)[\partial_{\alpha\dot\beta} + (1/2\mathrm{i})\mathscr{D}_\alpha(\exp(2V^t T^t)\bar{\mathscr{D}}_{\dot\beta}\exp(-2V^t T^t))]\exp(V^k T^k).\end{aligned} \tag{3.20}$$

Note that one more conventional constraint

$$F_{\alpha\dot\beta} = \mathscr{D}_\alpha \bar\Omega_{\dot\beta} + \bar{\mathscr{D}}_{\dot\beta}\Omega_\alpha + \mathrm{i}\{\Omega_\alpha, \bar\Omega_{\dot\beta}\} - 2\mathrm{i}\sigma^a_{\alpha\dot\beta}\Omega_a = 0 \tag{3.21}$$

is fulfilled identically in the present approach, by the definition (3.14).

Let us now define the three-index quantity

$$F_{\beta\rho\dot\alpha} = \nabla_\beta \Omega_{\rho\dot\alpha} - \mathrm{i}(\partial)_{\rho\dot\alpha}\Omega_\beta = \mathrm{i}\sigma^a_{\rho\dot\alpha}(\nabla_\beta\Omega_a - \partial_a\Omega_\beta) \equiv \mathrm{i}\sigma^a_{\rho\dot\alpha}F_{\beta a} \tag{3.22}$$

where ∇_β is the spinor gauge-covariant derivative in the adjoint representation of G,

$$\nabla_\beta = \mathscr{D}_\beta + \mathrm{i}[\Omega_\beta, \;\}, \tag{3.23}$$

and the commutator or anticommutator is chosen depending on whether the object on which ∇_β acts is even or odd. It is easy to see that under the transformations (3.3), (3.19) the strength (3.22) undergoes the homogeneous transformation

$$F'_{\beta a} = \exp(\mathrm{i}K^l T^l)F_{\beta a}\exp(-\mathrm{i}K^l T^l). \tag{3.24}$$

This strength and its conjugate $\bar F_{\dot\beta a}$ naturally arise when spinor gauge-covariant derivatives ∇_α, $\bar\nabla_{\dot\alpha}$ are commuted with the vector one ∇_a (3.11c):

$$[\nabla_\alpha, \nabla_a] = \mathrm{i}F_{\alpha a}, \qquad [\bar\nabla_{\dot\alpha}, \nabla_a] = \mathrm{i}\bar F_{\dot\alpha a}. \tag{3.25}$$

Using the relations (3.15) one may check that $F_{\alpha\beta\dot\rho}$ satisfies the equations

$$F_{\alpha\beta\dot\rho} = -F_{\beta\alpha\dot\rho}, \tag{3.26}$$

$$\nabla_\alpha F_{\beta\gamma\dot\rho} + \nabla_\beta F_{\alpha\gamma\dot\rho} = 0. \tag{3.27}$$

The first one implies that $F_{\alpha\beta\dot\rho}$ can be represented as

$$F_{\alpha\beta\dot\rho} = \tfrac{1}{2}\varepsilon_{\alpha\beta}\bar W_{\dot\rho} = \tfrac{1}{2}\varepsilon_{\alpha\beta}[\nabla^\gamma\Omega_{\gamma\dot\rho} - \mathrm{i}(\partial)_{\gamma\dot\rho}\Omega^\gamma]. \tag{3.28}$$

Then the second equation yields

$$\nabla_\alpha \bar{W}_{\dot\beta} = 0. \tag{3.29}$$

After passing to the right basis in the group space of G^c by the second of formulae (2.19)

$$\bar{W}^R_{\dot\alpha} = \exp(V^k T^k)\bar{W}_{\dot\alpha} \exp(-V^k T^k) \tag{3.30}$$

this condition reduces to the ordinary chirality condition:

$$\mathscr{D}_\alpha \bar{W}^R_{\dot\alpha} = \exp(V^k T^k)\nabla_\alpha \bar{W}_{\dot\alpha} \exp(-V^k T^k) = 0. \tag{3.31}$$

A direct calculation utilising the explicit expression for the form Ω_α, (3.8), gives

$$\bar{W}^R_{\dot\alpha} = (1/2\mathrm{i})\mathscr{D}^\alpha\mathscr{D}_\alpha(\exp(2V^tT^t)\bar{\mathscr{D}}_{\dot\alpha} \exp(-2V^tT^t)) \tag{3.32}$$

that coincides with the standard expression for the covariant spinor strength of the $N=1$ Yang–Mills theory. Using the relation (3.30) one easily establishes also the form of $\bar{W}_{\dot\alpha}$:

$$\bar{W}_{\dot\alpha} = (1/2\mathrm{i})\nabla^\alpha\nabla_\alpha(\bar\nabla_{\dot\alpha} \exp(-V^kT^k) \exp(V^kT^k)). \tag{3.33}$$

The tensor strength F_{ab} is defined through the commutator of vector gauge-covariant derivatives:

$$[\nabla_a, \nabla_b] = \mathrm{i}F_{ab} = \partial_a\Omega_b - \partial_b\Omega_a + \mathrm{i}[\Omega_a, \Omega_b]. \tag{3.34}$$

It is expressed in terms of W_α, $\bar{W}_{\dot\alpha}$ by the standard formula (Sohnius 1978)†

$$F_{ab} = \tfrac{1}{16}\mathrm{i}[\nabla^\alpha(\sigma_{ab})_\alpha{}^\beta W_\beta + \bar\nabla_{\dot\alpha}(\tilde\sigma^{ab})^{\dot\alpha}{}_{\dot\beta}\bar{W}^{\dot\beta}]. \tag{3.35}$$

Now we discuss couplings to matter. In conventional nonlinear realisations, the minimal interactions of matter fields with the Goldstone fields are introduced as follows. One starts with a Lagrangian invariant under the vacuum stability subgroup and then replaces the ordinary derivatives by the covariant ones. In our case, the vacuum stability subgroup is the group of rigid G-transformations. Therefore, in order to implement couplings between matter superfields themselves and with the prepotential V^k in the manner invariant under the whole group G^c_{loc}, it is sufficient to make the change $\mathscr{D}_\alpha$, $\bar{\mathscr{D}}_{\dot\alpha}$, $\partial_a \to \nabla_\alpha$, $\bar\nabla_{\dot\alpha}$, ∇_a in some superfield Lagrangian with global G symmetry. However, sometimes it is more convenient to bring superfields beforehand into the left or right complex G^c-bases with the help of the relations (2.19). All the geometric characteristics written above can be recast into these bases by formulae of the type (2.19):

$$\{\nabla^L_\alpha, \bar\nabla^L_{\dot\alpha}, \nabla^L_a\} = \exp(-V^kT^k)\{\nabla_\alpha, \bar\nabla_{\dot\alpha}, \nabla_a\} \exp(V^kT^k), \tag{3.36a}$$

$$\{\nabla^R_\alpha, \bar\nabla^R_{\dot\alpha}, \nabla^R_a\} = \exp(V^kT^k)\{\nabla_\alpha, \bar\nabla_{\dot\alpha}, \nabla_a\} \exp(-V^kT^k) \tag{3.36b}$$

(here, the derivatives are assumed to act on everything to the right of them). The explicit form of covariant derivatives in the left basis is as follows:

$$\nabla^L_\alpha = \mathscr{D}_\alpha + \mathrm{i}\mathscr{V}^{L^i}_\alpha T^i, \qquad \bar\nabla^L_{\dot\alpha} = \bar{\mathscr{D}}_{\dot\alpha}, \qquad \nabla^L_a = \partial_a + \mathrm{i}\tilde\sigma_a^{\dot\beta\alpha}\bar{\mathscr{D}}_{\dot\beta}\mathscr{V}^{L^i}_\alpha T^i \tag{3.37}$$

with $\mathscr{V}^{Li}_\alpha$ given by (3.7*a*). These operators are related to the corresponding quantities in the right basis by complex conjugation. The covariant strengths in the complex bases can be obtained directly by commuting relevant covariant derivatives between

† We use the standard definitions (see e.g. Ogievetsky and Mezincescu 1975) $\sigma_{ab} = (1/2\mathrm{i})(\sigma_a\tilde\sigma_b - \sigma_b\tilde\sigma_a)$, $\tilde\sigma_{ab} = (1/2\mathrm{i})(\tilde\sigma_a\sigma_b - \tilde\sigma_b\sigma_a) = (\sigma_{ab})^+$, $(\sigma_a)_{\alpha\dot\beta} = (I, \boldsymbol{\sigma})_{\alpha\dot\beta}$, $(\tilde\sigma_a)^{\dot\alpha\beta} = (I, -\boldsymbol{\sigma})^{\dot\alpha\beta}$.

themselves; they all are expressed through $\mathscr{V}^{\mathrm{L}}_{\alpha}$, $\bar{\mathscr{V}}^{\mathrm{R}}_{\dot\alpha}$, (3.7), and have a simpler appearance compared with those in the real basis (cf expressions (3.32) and (3.33)). Covariant derivatives $\bar\nabla^{\mathrm{L}}_{\dot\alpha}$, $\nabla^{\mathrm{R}}_{\alpha}$ do not contain dependence on $V^i(x,\theta,\bar\theta)$, so one may impose on Φ_{L}, Φ_{R} the ordinary chirality conditions

$$\bar{\mathscr{D}}_{\dot\alpha}\Phi^{\mathrm{I}}_{\mathrm{L}}=0\rightarrow\Phi^{\mathrm{I}}_{\mathrm{L}}=a^{\mathrm{I}}_{\mathrm{L}}(x_{\mathrm{L}},\theta_{\mathrm{L}}),\qquad \mathscr{D}_{\alpha}\Phi^{\mathrm{II}}_{\mathrm{R}}=0\rightarrow\Phi^{\mathrm{II}}_{\mathrm{R}}=a^{\mathrm{II}}_{\mathrm{R}}(x_{\mathrm{R}},\bar\theta_{\mathrm{R}}). \tag{3.38}$$

In the real basis, these constraints look more complicated:

$$\nabla_{\alpha}\Phi^{\mathrm{I}}=0\rightarrow\Phi^{\mathrm{I}}=\exp(V^i\bar{T}^i)a^{\mathrm{I}}_{\mathrm{L}}(x_{\mathrm{L}},\theta_{\mathrm{L}}),\qquad \bar\nabla_{\alpha}\Phi^{\mathrm{II}}=0\rightarrow\Phi^{\mathrm{II}}=\exp(-V^i\bar{T}^i)a^{\mathrm{II}}_{\mathrm{R}}(x_{\mathrm{R}},\bar\theta_{\mathrm{R}}). \tag{3.39}$$

Accordingly, one has two equivalent forms of the invariant kinetic term of chiral superfields (Siegel and Gates 1979, Grisaru 1981):

$$\mathscr{L}^{\mathrm{ch}}_{\mathrm{kin}}\sim\mathrm{Tr}(\Phi^{\mathrm{I}+}\Phi^{\mathrm{I}})=\mathrm{Tr}(a^{\mathrm{I}+}_{\mathrm{L}}\exp(2V^i\bar{T}^i)a^{\mathrm{I}}_{\mathrm{L}}). \tag{3.40}$$

3.3.

Now, let us discuss briefly the case of curved geometry on $\mathbb{R}^{4|4}$. We restrict our consideration to the standard minimal Einstein $N=1$ supergravity. To repeat the above analysis, one needs the following commutation relations between curved counterparts $\tilde{\mathscr{D}}_{\alpha}$, $\bar{\tilde{\mathscr{D}}}_{\dot\alpha}$, $\tilde{\mathscr{D}}_{a}$ of flat superspace derivatives (Wess and Zumino 1977, Grimm *et al* 1979, Ogievetsky and Sokatchev 1980b):

$$\{\tilde{\mathscr{D}}_{\alpha},\tilde{\mathscr{D}}_{\beta}\}=-R_{\alpha\beta},\qquad \{\tilde{\mathscr{D}}_{\alpha},\bar{\tilde{\mathscr{D}}}_{\dot\beta}\}=2\mathrm{i}\sigma^{a}_{\alpha\dot\beta}\tilde{\mathscr{D}}_{a}\equiv2\mathrm{i}\tilde{\mathscr{D}}_{\alpha\dot\beta}, \tag{3.41}$$
$$[\tilde{\mathscr{D}}_{\alpha},\tilde{\mathscr{D}}_{\beta\dot\beta}]=-T^{\gamma}_{\alpha,\beta\dot\beta}\tilde{\mathscr{D}}_{\gamma}-T^{\dot\gamma}_{\alpha,\beta\dot\beta}\bar{\tilde{\mathscr{D}}}_{\dot\gamma}-R_{\alpha,\beta\dot\beta}.$$

Here, the symbols T, R denote components of torsion and curvature (the latter takes values in the algebra sl$(2,c)$) and the conventional constraints are taken into account (we basically use the notation of Ogievetsky and Sokatchev (1980b)). For our purpose, it is necessary to know the explicit expressions of the components $R_{\alpha\beta,\gamma\delta}$, $T^{\dot\gamma}_{\alpha,\beta\dot\beta}$:

$$R_{\alpha\beta,\gamma\delta}=-\tfrac{1}{2}(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}+\varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma})\bar{R},\qquad T^{\dot\gamma}_{\alpha,\beta\dot\beta}=-\tfrac{1}{4}\mathrm{i}\varepsilon_{\alpha\beta}\delta^{\dot\gamma}_{\dot\beta}\bar{R}, \tag{3.42}$$

where $\bar{R}$ is one of the basic superfields of minimal $N=1$ supergravity. Also, we make use of one of the standard Bianchi identities (Grimm *et al* 1979)

$$R_{\alpha,\beta\dot\beta,\gamma\delta}+R_{\gamma,\beta\dot\beta,\alpha\delta}=-\tilde{\mathscr{D}}_{\alpha}T_{\gamma,\beta\dot\beta,\delta}-\tilde{\mathscr{D}}_{\gamma}T_{\alpha,\beta\dot\beta,\delta}. \tag{3.43}$$

All the basic gauge-covariant quantities of the flat case except for $F_{\beta\alpha\dot\alpha}$, (3.22), are generalised to the curved superspace simply by means of the change $\mathscr{D}_{\alpha}$, $\bar{\mathscr{D}}_{\dot\alpha}$, $\partial_a\rightarrow\tilde{\mathscr{D}}_{\alpha}$, $\bar{\tilde{\mathscr{D}}}_{\dot\alpha}$, $\tilde{\mathscr{D}}_{a}$ in the corresponding formulae. The strength $F_{\beta\alpha\dot\alpha}$ gets a minor modification

$$\tilde{F}_{\beta\alpha\dot\alpha}=\tilde\nabla_{\beta}\tilde\Omega_{\alpha\dot\alpha}-\mathrm{i}\tilde{\mathscr{D}}_{\alpha\dot\alpha}\tilde\Omega_{\beta}+\mathrm{i}T^{\gamma}_{\beta,\alpha\dot\alpha}\Omega_{\gamma}+\mathrm{i}T^{\dot\gamma}_{\beta,\alpha\dot\alpha}\bar{\tilde\Omega}_{\dot\gamma} \tag{3.44}$$

(that is owing to the non-zero torsion in the last of the relations (3.41)). Using (3.41)–(3.43), one may be convinced that $\tilde{F}_{\beta\alpha\dot\alpha}$ enjoys the same properties (3.26), (3.27), (3.29) as $F_{\beta\alpha\dot\alpha}$ in the flat case (of course, the change $\nabla_{\alpha}\rightarrow\tilde\nabla_{\alpha}$ should be made). A simple calculation yields for the spinor strength $\bar{\tilde{W}}_{\dot\alpha}=\varepsilon^{\alpha\beta}\tilde{F}_{\beta\alpha\dot\alpha}$ the well known expression (see e.g. Grisaru 1981)

$$\bar{\tilde{W}}_{\dot\alpha}=(1/2\mathrm{i})(\tilde\nabla^{\alpha}\tilde\nabla_{\alpha}+\bar{R})(\bar{\tilde\nabla}_{\dot\alpha}\exp(-V^kT^k)\exp(V^kT^k)) \tag{3.45}$$

which simplifies in the right G^c-basis to

$$\bar{\tilde{W}}^{R}_{\dot\alpha} = \exp(V^kT^k)\bar{\tilde{W}}_{\dot\alpha}\exp(-V^kT^k)$$
$$= (1/2i)(\tilde{\mathscr{D}}^\alpha\tilde{\mathscr{D}}_\alpha + \tilde{R})(\exp(2V^kT^k)\bar{\tilde{\mathscr{D}}}_{\dot\alpha}\exp(-2V^kT^k))$$
$$(\tilde{\mathscr{D}}_\alpha\bar{\tilde{W}}^{R}_{\dot\alpha} = \exp(V^kT^k)\tilde{\nabla}_\alpha\bar{\tilde{W}}_{\dot\alpha}\exp(-V^kT^k) = 0). \qquad (3.46)$$

Thus, we have demonstrated that all the relevant quantities of the $N = 1$ Yang–Mills theory can be obtained algorithmically starting solely with the structure relations (2.1) and the standard nonlinear realisation formulae (3.1) supplemented by the covariant constraints of the inverse Higgs phenomenon (3.6). Perhaps it would be interesting to relate this formalism to the Levi superform approach advocated by Schwarz (1981) as the most adequate geometric language to deal with hypersurfaces in complex superspaces (see also Gayduk *et al* 1981).

Finally, we note that with respect to the right gauge group (2.20) all the covariant objects in the real G^c-basis transform just as in $G^c_{\rm loc}$ but with arbitrary functions $\lambda^i(x, \theta, \bar\theta)$ instead of K^i. The corresponding quantities in the complex G^c-bases are invariant under this gauge group (it is checked with the help of the representation (2.22)). For these reasons, any Lagrangian invariant under $G^c_{\rm loc}$ turns out automatically invariant with respect to the right gauge G-transformations. The right gauge freedom can be used to represent all the basic quantities of the $N = 1$ Yang–Mills directly in terms of the $\mathbb{R}^{4|4}$ restrictions of original complex G^c-coordinates $\varphi^i_L(x, \theta, \bar\theta)$, $\varphi^i_R(x, \theta, \bar\theta)$. For instance, the forms Ω_α, $\bar\Omega_{\dot\alpha}$, (3.8), can be gauge transformed to

$$\check\Omega_\alpha = (1/i)\exp(-i\varphi^i_R T^i)\mathscr{D}_\alpha\exp(i\varphi^i_R T^i),$$
$$\check{\bar\Omega}_{\dot\alpha} = (1/i)\exp(-i\varphi^k_L T^k)\bar{\mathscr{D}}_{\dot\alpha}\exp(i\varphi^k_L T^k). \qquad (3.8')$$

4. Conclusion

The above consideration suggests several new interesting possibilities for the $N = 1$ Yang–Mills theory. First, the fact that this theory is a kind of the nonlinear σ model on the coset G^c/G raises the problem of constructing an appropriate linear σ model, with G^c as the vacuum stability group. Since any unitary representation of G^c is infinite dimensional (G^c is non-compact), such a σ model should naturally give rise to infinite-dimensional field multiplets. The standard $N = 1$ Yang–Mills theory would arise within this model dynamically, as a result of appearance of non-zero vacuum expectation values of certain components of the original linear multiplet breaking G^c-symmetry to G-symmetry. In fact, using general theorems on the relation between representations and nonlinear realisations (Coleman *et al* 1969), one may construct out of $V^i(x, \theta, \bar\theta)$ alone any representation of G^c including infinite-dimensional unitary ones, provided they contain an invariant of G. The possibility to construct such composite linear G^c-multiplets can be thought of as a group-theoretic argument in favour of the existence of the dynamical phase with unbroken G^c-symmetry in the $N = 1$ Yang–Mills theory. An interesting point is the inevitable presence of G-invariant (i.e. 'colourless') states in these multiplets.

Another line of thinking concerns the geometric analogy between the $N = 1$ Yang–Mills and $N = 1$ supergravity. A natural hypothesis is that these theories admit a unification within a larger theory of the Kaluza–Klein type. One may treat $\operatorname{Re}\varphi^i_L \equiv \varphi^i$ as an independent coordinate like x^m in (1.1), choose the base superspace to be $\mathbb{R}^{4+M|4} = \{x^m, \varphi^i, \theta^\mu, \bar{\theta}^{\dot{\mu}}\}$ instead of $\mathbb{R}^{4|4}$, and construct a $(4+M)$-dimensional extension of minimal $N=1$ supergravity by embedding $\mathbb{R}^{4+M|4}$ into $\mathbb{C}^{4+M|2}$. The standard theory is expected to be reproduced as the lowest order in a proper expansion in φ^i.

However, the most exciting and urgent task is to extend the geometric picture described here to higher-N gauge theories, at least to the case of $N=2$. The necessity to complexify G in the $N=1$ case can be related to the fact that the fundamental superspace of $N=1$ supersymmetry is the complex superspace $\mathbb{C}^{4|2}$. Its true analogue for $N=2$ seems to be a superspace, bosonic coordinates of which form a quaternion (Galperin *et al* 1981, 1982). So, in the $N=2$ case one may, instead of the extension $T^k \to \{T^k, \mathrm{i}T^k\}$, try an extension of the type $T^k \to \{T^k, q^i \otimes T^k, \ldots\}$ where q^i ($i = 1, 2, 3$) are imaginary quaternion units transforming as a triplet with respect to the automorphism group SU(2) of the $N=2$ superalgebra. The relevant prepotential should then acquire an additional triplet index. That is just what happens in the $N=2$ electrodynamics (Mezincescu 1979). Work along these lines is now in progress. We believe that the elucidation of the intrinsic superspace geometric structure of the $N=2$ Yang–Mills theory will essentially help in understanding the analogous structure of $N=2$ supergravity.

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Note added in proof. An interpretation of the $N=1$ Yang–Mills prepotential analogous to that suggested by Ivanov (1982) has been independently given also by Rosly in the recent paper: Rosly A A 1982 *J. Phys. A: Math. Gen.* **15** L663.

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